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Quantum Teleportation of an Unknown Two-Atom Entangled State Using Four-Atom Cluster State

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Abstract A simply protocol for quantum teleportation of an unknown two-atom entangled state using four-atom cluster state is investigated in cavity quantum electrodynamics (QED). In this protocol, by using a one-dimensional maximally four-atom cluster state as quantum channel, an unknown two-atom entangled state can be transmitted from the sender (Alice) to the receiver without apparent joint Bell-state measurement. According to the results measured by the sender, the receiver can obtain the original state with unit successful probability. The important features of our scheme can also be demonstrated in ion trap system.

Keywords Quantum teleportation · Four-atom cluster state · Cavity quantum electrodynamics

1 Introduction

Quantum entanglement, one of the most interesting features in quantum mechanics, plays a significant role in quantum information processing [1–3]. Quantum teleportation, originally proposed by Bennett et al., is a technique for transferring quantum states from a sender (Alice) to a receiver (Bob) [4]. Since the theoretical protocol for teleportation of a single qubit, many protocols of quantum teleportation are proposed and have been demonstrated experimentally by several groups [5–8]. With the development of quantum physics, the information techniques based on quantum physics have been actively studied. Recently, more and

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more attentions are paid to the study of quantum teleportation with many-particle entangled state. In 1998, Karlsson and Bourennane [9] presented a protocol of teleporting two qubits state using GHZ state instead of an EPR pair. Xiu et al. [10] proposed an teleportation protocol of an N-particle state via three-particle general W states. Lu [11] proposed a protocol for the teleportation of two-particle entangled state via entanglement swapping. In above quantum teleportation schemes, the entangled states as quantum channel are EPR pairs, GHZ class states, or W class states. Tripartite entangled states can be classified into two inequivalent classes, the Greenberger-Horne-Zeilinger (GHZ) class and the W class. While, the four-particle entangled state was divided to different families of states under stochastic local operations and classical communication (SLOCC). Recently, Briegel and Raussendorf introduced an interesting type of multi-qubit entangled states, i.e., the so-called cluster states [12]. This kind of states have high persistence of entanglement, and can be regarded as an entanglement resource for the GHZ states but are more immune to decoherence than them. The cluster states have extensive applications in quantum physics. Many theoretical schemes of generating cluster states have been proposed in different types of physical systems, such as linear optical systems [13], cavity QED [14], and other kinds of systems [15]. Experimentally, Mandel et al. [16] prepare the cluster state of neutral atom in optical lattice. Walther et al. [17] have generated four-photon cluster states and demonstrated the feasibility of the one-way quantum computation. On the other hand, the microwave cavity QED, with Rydberg atoms crossing superconducting cavities, provides an almost ideal system for the realization of quantum information processing. In cavity QED, schemes have been proposed for teleportation of an unknown state. Davidovich et al. [18] have presented a scheme for the teleportation of an unknown atomic state between two high-Q cavities. Zheng [19] has proposed a scheme for teleporting an unknown atomic state in cavity QED. Jin et al. [20] made a proposal for teleporting two-atom state with a probability 1.0 by adding a classical driving field.

In this paper, we propose a scheme for the teleportation of an unknown two-atom two-level entangled state in driven cavity QED. In contrast to the previous scheme, the present one has the following advantages: First, such an unknown two-atom state is transmitted from a sender to a receiver. Second, the quantum channel is different, in our protocol, the quantum channel is composed of four-atom cluster state. Whereas generating cluster states have been proposed in different types of physical systems. Finally the protocol does not involve apparent (or direct) Bell-state measurement and is insensitive to the cavity decay and the thermal field. The probability of the success can reach 1.0. Meanwhile, the important features of our scheme can also be demonstrated in ion trap system.

2 The Teleportation of an Unknown Two-Atom Entangled State

Assume the two atoms that Alice wants to teleport are initially in the state

$$|\phi\rangle_{12} = a|gg\rangle_{12} + b|ge\rangle_{12} + c|eg\rangle_{12} \tag{1}$$

where a, b, and c are unknown coefficients, $|a|^2 + |b|^2 + |c|^2 = 1$. $|e\rangle$ and $|g\rangle$ are the excited and ground states of the atom. The quantum channel which shared by Alice and Bob is a one-dimensional maximally four-atom cluster state (Atoms 3, 4 5, 6) which given by

$$|\phi\rangle_{3456} = \frac{1}{2}(|gggg\rangle_{3456} + |eegg\rangle_{3456} + |ggee\rangle_{3456} - |eeee\rangle_{3456}) \tag{2}$$

where the sender Alice has the atoms 1, 3 and 2, 5 and the other atoms 4, 6 belong to the receiver Bob.

The initial state of the whole system is given by

$$|\psi\rangle_{123456} = \frac{1}{2}(a|gg\rangle_{12} + b|ge\rangle_{12} + c|eg\rangle_{12}) \otimes (|gggg\rangle_{3456} + |eegg\rangle_{3456} + |ggee\rangle_{3456} - |eeee\rangle_{3456}) \tag{3}$$

In order to help Bob to realize teleportation, Alice sends atoms 1, 3 into a single-mode cavity and atoms 2, 5 into another cavity. At the same time, the two atoms are driven by a classical field. The interaction between atoms and the cavity can be described as follows [21]

$$H = \omega_0 \sum_{j=1}^{2}(S_j^z) + \omega_a a^\dagger a + \sum_{j=1}^{2}[g(a^\dagger S_j^- + aS_j^+) + \Omega(S_j^+ e^{-i\omega_d t} + S_j^- e^{i\omega_d t})] \tag{4}$$

where ω_0, ω_a and ω_d are atomic transition frequency, cavity frequency and the frequency of driving field, respectively, $a^\dagger$ and a are creation and annihilation operators for the cavity mode, g is the coupling constant between atoms and cavity, atomic operators $S_j^+ = |e\rangle_j\langle g|$, $S_j^- = |g\rangle_j\langle e|$, $S_j^z = \frac{1}{2}(|e\rangle_j\langle e| - |g\rangle_j\langle g|)$, Ω is the Rabi frequency of the classical field. We consider the case $\omega_0 = \omega_d$. In the interaction picture, the evolution operator of the system is[20]

$$U(t) = e^{-iH_0 t} e^{-iH_e t} \tag{5}$$

where $H_0 = \sum_{j=1}^{2} \Omega(S_j^- + S_j^+)$, H_e is the effective Hamiltonian. In the large detuning $\delta \gg \frac{1}{2}g$ and strong driving field, $2\Omega \gg \delta, g$ limit, the effective Hamiltonian for this interaction can be described as follows:

$$H_e = \lambda\left[\sum_{j=1}^{2}\frac{1}{2}(|e\rangle_j\langle e| + |g\rangle_j\langle g|) + \sum_{j,k=1, j\neq k}^{2}(S_j^+ S_k^+ + S_j^+ S_k^-) + H.c\right] \tag{6}$$

where $\lambda = g^2/2\delta$, δ is the detuning between ω_0 and ω_a. From (6), we know H_e is independent of creation and annihilation operators of the cavity mode and only have relation with atomic operators. So the effects of cavity decay and thermal field are all eliminated. We can choose $\lambda t = \frac{1}{4}\pi$, $\Omega t = \pi$ by modulating the driving field appropriately. Hence the total state of the combined system is

$$\begin{aligned}
|\psi\rangle = \frac{1}{4}[&|eeee\rangle_{1325} \times (-a|gg\rangle_{46} - ib|ge\rangle_{46} - ic|eg\rangle_{46}) \\
&+ |eegg\rangle_{1325} \times (-ia|gg\rangle_{46} - b|ge\rangle_{46} + c|eg\rangle_{46}) \\
&+ |ggee\rangle_{1325} \times (-ia|gg\rangle_{46} + b|ge\rangle_{46} - c|eg\rangle_{46}) \\
&+ |gggg\rangle_{1325} \times (a|gg\rangle_{46} - ib|ge\rangle_{46} - ic|eg\rangle_{46}) \\
&+ |egeg\rangle_{1325} \times (a|ee\rangle_{46} - ib|eg\rangle_{46} - ic|ge\rangle_{46}) \\
&+ |egge\rangle_{1325} \times (ia|ee\rangle_{46} - b|eg\rangle_{46} + c|ge\rangle_{46}) \\
&+ |geeg\rangle_{1325} \times (ia|ee\rangle_{46} + b|eg\rangle_{46} - c|ge\rangle_{46}) \\
&+ |gege\rangle_{1325} \times (-a|ee\rangle_{46} - ib|eg\rangle_{46} - ic|ge\rangle_{46})
\end{aligned}$$

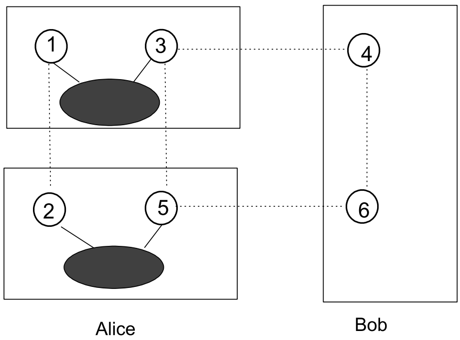

Fig. 1 The teleportation of an unknown two-atom entangled state. The *hollow circle* stands for an atom. The *dot line* between two atoms represents their entanglement. The *ellipse* stands for a cavity. See text for detail

$$
\begin{aligned}
&+|eeeg\rangle_{1325}\times(-a|ge\rangle_{46}-ib|gg\rangle_{46}+ic|ee\rangle_{46})\\
&+|eege\rangle_{1325}\times(-ia|ge\rangle_{46}-b|gg\rangle_{46}-c|ee\rangle_{46})\\
&+|ggeg\rangle_{1325}\times(-ia|ge\rangle_{46}+b|gg\rangle_{46}+c|ee\rangle_{46})\\
&+|ggge\rangle_{1325}\times(a|ge\rangle_{46}-ib|gg\rangle_{46}+ic|ee\rangle_{46})\\
&+|egee\rangle_{1325}\times(-a|eg\rangle_{46}+ib|ee\rangle_{46}-ic|gg\rangle_{46})\\
&+|eggg\rangle_{1325}\times(-ia|eg\rangle_{46}+b|ee\rangle_{46}+c|gg\rangle_{46})\\
&+|geee\rangle_{1325}\times(-ia|eg\rangle_{46}-b|ee\rangle_{46}-c|gg\rangle_{46})\\
&+|gegg\rangle_{1325}\times(a|eg\rangle_{46}+ib|ee\rangle_{46}-ic|gg\rangle_{46})]
\end{aligned}
\tag{7}
$$

In order to realize the teleportation, Alice should make a separate measurement on atoms 1, 3, 2 and 5. Without lose of generality, if Alice detects the atoms in the state $|eeee\rangle_{1325}$, the state of atoms 4 and 6 will collapse into

$$-a|gg\rangle_{46}-ib|ge\rangle_{46}-ic|eg\rangle_{46} \tag{8}$$

where we have discarded the global phase factor.

Now Alice informs Bob of the result of the measurement by the classical channels, then Bob performs corresponding single-atom rotations on atoms 4 and 6 ($|g\rangle\to-|g\rangle, |e\rangle\to i|e\rangle)\otimes(|g\rangle\to|g\rangle, |e\rangle\to i|e\rangle$) to recover the teleported state, (8) will become

$$a|gg\rangle_{46}+b|ge\rangle_{46}+c|eg\rangle_{46} \tag{9}$$

Now the teleportation is successful. From (7), we can see that when atoms 1, 3 and atoms 2, 5 enter into the cavity, because every atom has two levels, 16 kinds of different separate states can be derived. It is evident that Bob must operate relevant unitary transformation against Alice different measurement results. By the similar method, if the measurement results are $|eegg\rangle_{1325}$, $|ggee\rangle_{1325}$, $|gggg\rangle_{1325}$, $|egeg\rangle_{1325}$, $|egge\rangle_{1325}$, $|geeg\rangle_{1325}$, $|gege\rangle_{1325}$, $|eeeg\rangle_{1325}$, $|eege\rangle_{1325}$, $|ggeg\rangle_{1325}$, $|ggge\rangle_{1325}$, $|eggg\rangle_{1325}$, $|egee\rangle_{1325}$, $|geee\rangle_{1325}$ and $|gegg\rangle_{1325}$, the teleportation also can succeed with the equal probabilities. Now we calculate the probability of successful teleportation in this scheme. From (7) to (8), the probability of detecting the state $|eeee\rangle_{1326}$ is $1/16$, the probabilities of successful teleportation in the other fifteen measurement results are easily derived. That is to say, the total success probability is $1/16\times16=1.0$, and the fidelity of the output state is 1.0. Finally, it is necessary to give a brief discussion on the experimental matters. We consider the typical experimental values of the parameters for Rydberg atoms with principal quantum numbers 49, 50, 51, the radiative time is about $T_r = 3 \times 10^{-2}$ s, and the coupling constant is $g = 2\pi \times 24$ kHz [22]. For a normal cavity, the decay time can reach $T_c = 1.0 \times 10^{-3}$ s. Then we get that the interaction time of atom and cavity is on the order of 10^{-4} s. Hence the total time for the whole system is much shorter than T_r and T_c so that the present scheme might be realizable based on cavity QED techniques.

3 Summary

In conclusion, we have proposed a simple scheme to realize the teleportation of an unknown two-atom state. Two atoms exist in single-mode cavity and are driven by a classical field. Our schemes use a cluster state as the quantum channel and do not need any joint BSM. The schemes are realized by utilizing the separated atomic measurements instead of joint BSM, which reduce the complexity in experiment. During the passage of the atoms through the cavity field, the cavity is only virtually excited, and so the scheme is free of the cavity decay and the thermal field. Meanwhile the success probability is equal to 1. Generating cluster states have been proposed in different types of physical systems, thus this scheme is realizable by current available technology. The important features of our scheme can also be demonstrated in ion trap system. This may open a new perspective for the applications of cluster states in quantum information processing.

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